FIGURES

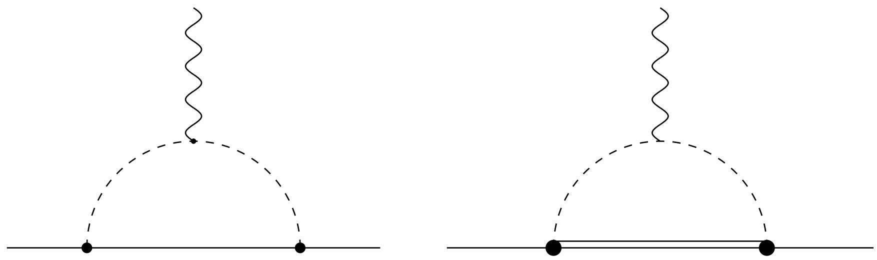

FIG. 1. Loop diagrams contributing to magnetic moments at $\mathcal{O}(1/\Lambda_\chi^2)$. The single internal line denotes an intermediate octet state while the double line denotes a decuplet state.

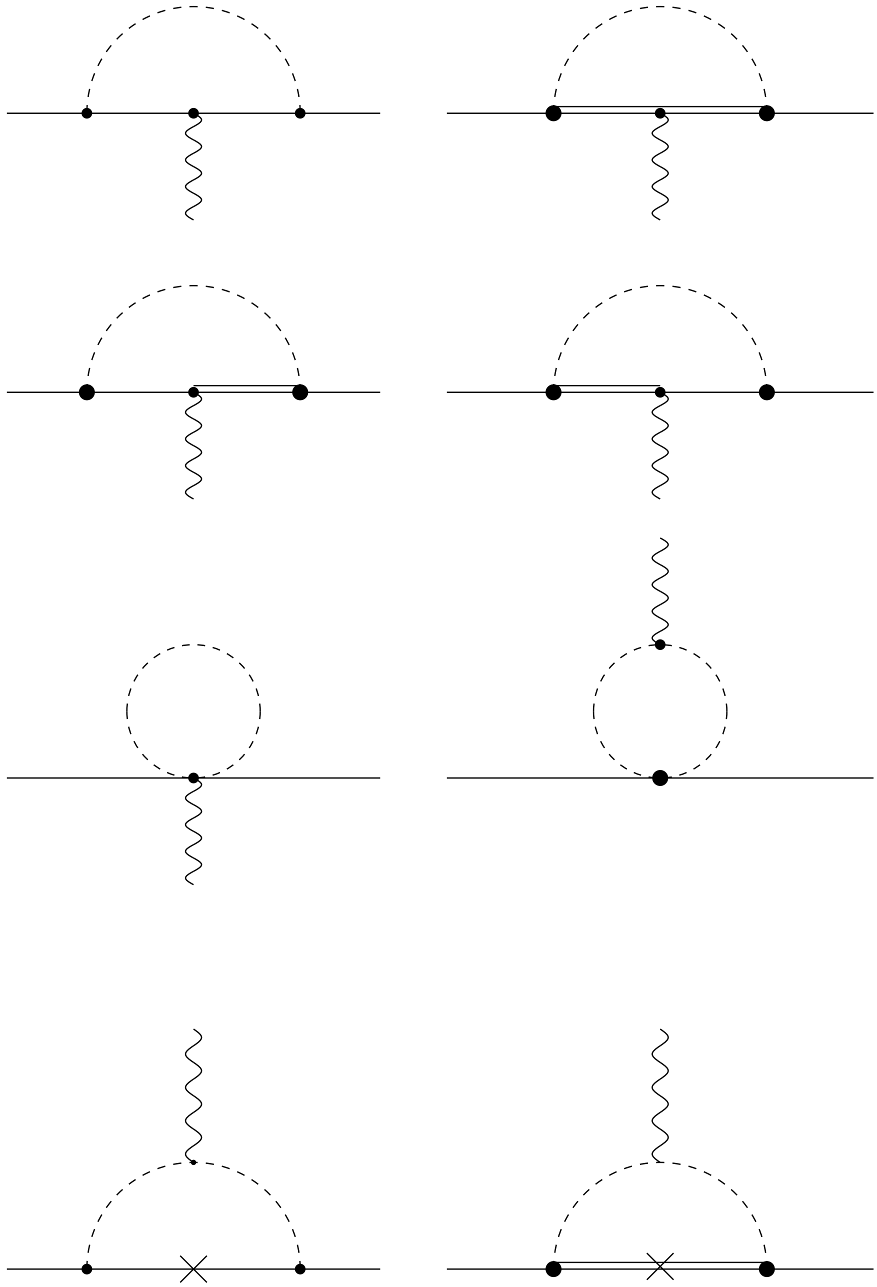

FIG. 2. Loop diagrams contributing to the magentic moments at $\mathcal{O}(1/\Lambda_\chi^3)$. The "$\times$" denotes $\mathcal{O}(1/M_N)$ vertex.

Baryon Octet magnetic moments in χPT: More on the importance of the Decuplet

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Abstract

We address the impact of treating the decuplet of spin-$\frac{3}{2}$ baryons as an explicit degree of freedom in the chiral expansion of the magnetic moments of the octet of spin-$\frac{1}{2}$ baryons. We carry out a complete calculation of the octet moments to $\mathcal{O}(1/\Lambda_\chi^3)$, including decuplet contributions to the chiral loops. In contrast to results of previous analyses, we find that inclusion of the decuplet preserves the convergence behavior of the chiral expansion implied by power counting arguments.

The application of Heavy Baryon Chiral Perturbation Theory (HBχPT) to low energy baryon properties has yielded considerable insight. For example, baryon masses, Compton scattering amplitudes, nucleon polarizabilities, sigma terms, axial couplings, and hyperon decays have all been investigated, [1]. To a large extent, a consistent description based on chiral symmetry has emerged. The electromagnetic (EM) properties of baryons have also been studied, with particular emphasis on the magnetic moments of the lowest-lying baryon octet ([2], [3], [4]). In contrast to the situation with other baryon properties, the success of an HBχPT description of these moments has been debated. The point of controversy has been whether the chiral expansion of the octet moments behaves as one would naïvely expect, based on power counting arguments. Specifically, if μ_B is a generic octet magnetic moment, one expects its chiral expansion to go schematically as

$$\mu_b = \mu_B^{(0)} + \mu_B^{(1)}\left(\frac{p}{\Lambda_\chi}\right) + \mu_B^{(2)}\left(\frac{p}{\Lambda_\chi}\right)^2 + \cdots \quad , \tag{1}$$

where $\mu_B^{(0)}$ is the tree-level magnetic moment, p is of order the pseudoscalar meson masses, $\Lambda_\chi = 4\pi f_\pi \approx 1$ GeV, and the $\mu_B^{(n)}$, $n > 1$, represent the long-distance (loop) and short-distance (counterterm) corrections to $\mu_B^{(0)}$ at a given order. To the extent that the $\mu_B^{(n)}$ are all of a similar order of magnitude, the relative size of successive terms in the expansion of Eq. (1) should decrease by the corresponding power of (p/Λ_χ). Taking $p \sim m_K$, for example, the expansion parameter should be or order $m_K/\Lambda_\chi \sim 1/2$.

The degree to which this behavior holds for the magnetic moments has been debated, and various remedies have been proposed for the apparent deviation of the expansion from this expectation. These remedies include adjusting the size of tree-level axial couplings to reduce the scale of kaon loop contributions [2], inclusion of $1/M_N$ corrections and higher-derivative terms [3], explicit retention of the leading analytic (in quark mass) loop contributions [3], and inclusion of the decuplet [4]. To date, no $\mathcal{O}(q^4)$ calculation has included both octet and decuplet loop contributions as well as the leading $1/M_N$ corrections and two-derivative operators. In an attempt to resolve some of the controversy, we have performed such a calculation. We find that that when decuplet is included as an explicit degree of freedom – along with the leading $1/M_N$ corrections and two-derivative contributions – the magnetic moment expansion behaves according to naïve power counting expectations (Eq. (1)) *without* including analytic loop contributions or adjusting the size of tree-level axial couplings.

Before discussing our calculation in detail, we review the history of HBχPT magnetic moment analyses. Jenkins and Manohar [6] note that due to the strength of the decuplet-octet coupling $\mathcal{C}$, and the small size of the mass splitting δ (relative to the intrinsic hadronic scale), the contributions from the decuplet should be larger than those from higher baryon resonances and comparable to the octet contribution. In the case of the axial current [7] substantial cancellations occur. It is then surprising that in the $\mathcal{O}(q^4)$ magnetic moment calculation of Ref. [2], inclusion of the decuplet does not produce appreciably better agreement with the data than the case where only the octet was included. The authors of Ref. [2] argue that there is some evidence that χPT overestimates the size of the kaon loops. They propose compensating for this effect by using the smaller one-loop corrected axial couplings in the calculation rather than the tree-level values.

The study of Ref. [3], however, suggests that the calculation of [2] is incomplete. In particular, it neglects the $1/M_N$ corrections and the contributions of certain double derivative operators, both of which occur at $\mathcal{O}(q^4)$. The decuplet was not included explicitly in the analysis of Ref. [3]. Its effect was only considered in determining its contribution to some low energy constants (LEC). These LEC's appear as couplings to operators of the form $\partial_\mu\phi\partial_\nu\phi$, where ϕ denotes the Goldstone boson field, which contribute at $\mathcal{O}(q^4)$. The same LEC's also receive contributions from the vector mesons and higher lying resonances such as the Roper octet. By expanding decuplet loop amplitudes in powers of $1/\delta$, the authors of Ref. [3] argue that the LEC's in question contain the leading order decuplet contribution. They conclude that there is no need to employ smaller values for the axial couplings or to include the decuplet explicitly. They also conclude from their calculation that the chiral expansion converges as expected. We note, however, that Ref. [3] retains contributions analytic in the quark mass which arise from loop contributions at $\mathcal{O}(q^4)$.

The later analysis of Durand and Ha [4] re-examines the analysis of Ref. [2], and focuses mainly on the convergence of the expansion. These authors argue that the decuplet must be included explicitly. They observe that the treatment of Ref. [3] requires the decuplet-octet mass splitting to be large with respect to the momentum in the loop integrals, which is not the case. They conclude that since the mass splittng is approximately 300 MeV, the decuplet must be considered as "light" and included explicitly. A similar observation appears in the work of Banerjee *et al.* [8]. The authors of Ref. [4] conclude that the chiral expansion for

the magnetic moments of the octet is not convergent. However, they did not include all of the terms contributing at $\mathcal{O}(q^4)$.

In what follows, we attempt to resolve this controversy. We do so by carrying out the a complete analysis of the octet magnetic moment at $\mathcal{O}(q^4)$ by including the decuplet explicitly and the full set of $1/M_N$ corrections. Our analysis is similar in spirit to that of Ref. [3], but differs in two respects: (a) the explicit inclusion of the decuplet, and (b) retention of only non-analytic loop contributions.

To make our notation and conventions clear, we review some elements of the HBχPT formalism. In this formalism a consistent chiral expansion of the baryon Lagrangian can be written in terms of the velocity-dependent octet and decuplet fields:

$$B_v(x) = \exp(iM_B \not{v} \cdot x)B(x);\ T_v^\mu(x) = \exp(iM_B \not{v} \cdot x)T^\mu(x) \quad , \tag{2}$$

here $B(x)$ and $T^\mu(x)$ denote the baryon octet and decuplet fields, respectively, and M_B is the SU(3) invariant mass of the octet. Defining the fields in this way eliminates ambiguities in power counting that arise due to the introduction of another large mass scale in the theory, *i.e.* the octet mass [6].

The leading order contributions to the magnetic moments of the octet are calculated from tree level graphs with vertices from the Lagrangians (we use the notation of [5]):

$$\mathcal{L}_1 = \frac{e}{\Lambda_\chi}\,\epsilon_{\mu\nu\rho\sigma}v^\rho \left\{ b_+ \ \mathrm{Tr}\left(\bar{B}_v S_v^\sigma \{Q, B_b\}\right) + b_- \ \mathrm{Tr}\left(\bar{B}_v S_v^\sigma [Q, B_v]\right)\right\} F^{\mu\nu} \tag{3}$$

Here we have introduced the covariant spin operator S_v^μ whose properties are discussed in Ref. [6]. We choose to normalize in powers of $1/\Lambda_\chi$ ($\Lambda_\chi = 4\pi f_\pi \approx 1\mathrm{GeV}$). In what follows, we count in powers of $1/\Lambda_\chi$ rather than in powers of q as is done in Ref. [3] (for instance, the conversion $\mathcal{O}(q^4) \leftrightarrow \mathcal{O}(1/\Lambda_\chi^3)$applies).

One-loop corrections are generated using the vertices from the lowest order chiral Lagrangian for octet and decuplet baryons which depends on the octet of pseudoscalar mesons $\tilde{\Pi}$. Introducing the the non-linear representation of the mesons

$$\xi = e^{i\tilde{\Pi}/f_\pi} \tag{4}$$

$$\Sigma = \xi^2 \tag{5}$$

and defining the vector and axial vector combinations

$$V_\mu \equiv \frac{1}{2}(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger) \tag{6}$$

$$A_\mu \equiv \frac{i}{2}(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger) \quad , \tag{7}$$

we write the lowest order Lagrangian (using the notation of [2])

$$\begin{aligned}\mathcal{L}_0 = {} & i \ \mathrm{Tr}\left(\bar{B}_v v\cdot D\ B_v\right) + 2D \ \mathrm{Tr}\left(\bar{B}_v S_v^\mu\{A_\mu, B_v\}\right) \\ & +2F \ \mathrm{Tr}\left(\bar{B}_v S_v^\mu [A_\mu, B_v]\right) - i\bar{T}_v^\mu \left(v\cdot\mathcal{D}\right) T_{v\mu} \\ & +\delta \bar{T}_v^\mu T_{v\mu} + \mathcal{C}\left(\bar{T}_v^\mu A_\mu B_v + \bar{B}_v A_\mu T_{v\mu}\right) + 2\mathcal{H}\bar{T}_v^\mu S_v^\nu A_\nu T_{v\mu} + \frac{{f_\pi}^2}{4}\mathrm{Tr}\left(\partial^\mu\Sigma^\dagger\partial_\mu\Sigma\right).\end{aligned} \tag{8}$$

Here $\delta = M_T - M_B$ is the baryon octet-decuplet mass splitting which arises due to the way we defined the velocity dependent fields. We use $D = .75$, $F = .50$ and $\mathcal{C} = -1.5$ throughout [3]. The value of $\mathcal{H}$ is not needed here. Interactions due to the vector current V_μ appear in the chiral covariant derivatives

$$D_\mu B = \partial_\mu B + [V_\mu, B]$$

and

$$\mathcal{D}_\nu T^\mu_{ijk} = \partial_\nu T^\mu_{ijk} + (V_\nu)^l_i T^\mu_{ljk} + (V_\nu)^l_j T^\mu_{ilk} + (V_\nu)^l_k T^\mu_{ijl}$$

where $i, j, k = 1, 2, 3$ are SU(3) flavor indices.

The electromagnetic interaction is incorporated into $\mathcal{L}_0$ via the substitutions

$$V_\mu \to V_\mu + \frac{1}{2} ie\mathcal{A}_\mu \left(\xi^\dagger Q\xi + \xi Q\xi^\dagger\right) \tag{9}$$

$$A_\mu \to A_\mu - \frac{1}{2} e\mathcal{A}_\mu \left(\xi^\dagger Q\xi - \xi Q\xi^\dagger\right) \tag{10}$$

and

$$\partial_\mu \Sigma \to \partial_\mu \Sigma + \frac{1}{2} ie\mathcal{A}_\mu [Q, \Sigma] \ , \tag{11}$$

where $\mathcal{A}_\mu$ is the photon field. The full chiral structure of the Lagrangian $\mathcal{L}_1$ is given by the replacement

$$Q \to \frac{1}{2}\left(\xi^\dagger Q\xi + \xi Q\xi^\dagger\right) \quad . \tag{12}$$

For the magnetic moments to $\mathcal{O}(1/\Lambda_\chi^3)$ at one-loop there are further contributions. First there are insertions of the leading order moments into the loops as well as insertions of the decuplet magnetic moment and the octet-decuplet transition moments. The decuplet magnetic moment operator can be written

$$\mathcal{L}_{\mathrm{T}} = -ie \ \frac{\tilde{\mu}_{\mathrm{C}} q_i}{\Lambda_\chi} \bar{T}^\mu_{vi} T^\nu_{vi} F_{\mu\nu} \ , \tag{13}$$

where q_i is the charge of the ith member of the decuplet. The measured value of the Ω^- moment determines $\tilde{\mu}_{\mathrm{C}} = 1.20 \pm 0.14$ (in our normalization). The octet-decuplet transition operator is given by ([2] and references therein)

$$\mathcal{L}_{\mathrm{BT}} = ie \ \frac{\tilde{\mu}_{\mathrm{T}}}{\Lambda_\chi} \left(\epsilon_{ijk} Q^i_l \bar{B}^j_m S^\mu_v T^{\nu klm} + \epsilon^{ijk} Q^l_i \bar{T}^\mu_{lkm} S^\nu_v B^m_j\right) F_{\mu\nu} \ , \tag{14}$$

where $i, j, k, l, m = 1, 2, 3$ are flavor indices. Measured values for $\Delta \to \gamma N$ helicity amplitudes determine $\tilde{\mu}_{\mathrm{T}} = -4.79 \pm 0.31$ (again in our normalization).

There is an additional set of dimension five operators which generates the double derivative operators mentioned above. They contribute to loops at $\mathcal{O}(1/\Lambda_\chi^3)$ [3] and are given by

$$\mathcal{L}_{\rm MB} = \frac{4i}{\Lambda_\chi}\,\epsilon_{\mu\nu\rho\sigma}v^\rho\left\{b_9\ \mathrm{Tr}(\bar{B}_v S_v^\sigma A^\mu)\ \mathrm{Tr}\,(A^\nu B_v) + b_{10}\ \mathrm{Tr}\left(\bar{B}_v S_v^\sigma\,[A^\mu, A^\nu]\,B_v\right)\right.$$
$$\left. + b_{11}\ \mathrm{Tr}\left(\bar{B}_v S_v^\sigma\{A^\mu, A^\nu\}B_v\right)\right\}. \qquad (15)$$

The loops derived from the operators listed above generate $\mathcal{O}(1/\Lambda_\chi^2)$ and $\mathcal{O}(1/\Lambda_\chi^3)$ contributions. Additional contributions of $\mathcal{O}(1/\Lambda_\chi^2 M_N)$ are obtained from the $1/M_N$ expansion of the lowest order Lagrangian. Only the corrections to the baryon propagators contribute and are given by [9]

$$\mathcal{L}_{\frac{1}{M_N}} = \frac{1}{2M_N}\left\{\ \mathrm{Tr}\left(\bar{B}_v[v\cdot D,[v\cdot D, B_v]]\right) - \ \mathrm{Tr}\left(\bar{B}_v[D^\mu,[D_\mu, B_v]]\right)\right.$$
$$\left. + \ \bar{T}_v^\mu\left(\mathcal{D}^\alpha\mathcal{D}_\alpha - v\cdot\mathcal{D}\ v\cdot\mathcal{D}\right)T_{v\mu}\right\}. \qquad (16)$$

Finally, along with the above couplings, the calculation of the magnetic moments requires the introduction of counter terms which break chiral SU(3) symmetry at $\mathcal{O}(1/\Lambda_\chi^3)$.

$$\mathcal{L}_{\rm SB} = \frac{e}{\Lambda_\chi}\,\epsilon_{\mu\nu\rho\sigma}v^\rho F^{\mu\nu}\left\{b_3\ \mathrm{Tr}\left(\bar{B}_v S_v^\sigma\,[[Q,B],\mathcal{M}]\right) + b_4\ \mathrm{Tr}\left(\bar{B}_v S_v^\sigma\,\{[Q,B],\mathcal{M}\}\right)\right.$$
$$\left. + b_5\ \mathrm{Tr}\left(\bar{B}_v S_v^\sigma\,[\{Q,B\},\mathcal{M}]\right) + b_5\ \mathrm{Tr}\left(\bar{B}_v S_v^\sigma\,\{\{Q,B\},\mathcal{M}\}\right) + b_7\ \mathrm{Tr}\left(\bar{B}_v S_v^\sigma B\right)\ \mathrm{Tr}\,(\mathcal{M}Q)\right\}. \quad (17)$$

Chiral SU(3)-breaking is introduced through the strange quark mass by the matrix $\mathcal{M} = B_0 m_s \mathrm{diag}(0,0,1)$, where B_0 carries dimensions of mass and is related to the scalar quark condensate. Since $\mathcal{M}$ counts as two powers of the meson mass it appears that our normalization for these symmetry breaking terms is incorrect. This is not the case. It might seem more natural to write the coupling and normalization as

$$\frac{B_0 m_s \tilde{b}_i}{\Lambda_\chi^3} \qquad (i = 3-7)$$

with the $\tilde{b}_i$'s of $\mathcal{O}(1)$. However, we avoid taking explicit values for B_0 and m_s by absorbing these and two powers of Λ_χ into our couplings, thus writing

$$b_i = \frac{B_0 m_s \tilde{b}_i}{\Lambda_\chi^2}$$

so that our normalization follows. With this choice, all the tree graphs appearing in the calculation of the magnetic moment appear to be of $\mathcal{O}(1/\Lambda_\chi)$. However, they contribute at different orders and now the couplings for the symmetry breaking terms are no longer of natural size.

Using the above conventions, we compute the EM magnetic moments to $\mathcal{O}(1/\Lambda_\chi^3)$ as generated by the tree-level operators of Eqs.(3, 17) and the non-analytic contributions from

the one-loop graphs of Fig. 1. Following a similar notation to that of [4] we write the results for the magnetic moments as

$$\mu_B = \left(\frac{2M_N}{\Lambda_\chi}\right)\Bigg\{\alpha_B + \frac{\pi}{\Lambda_\chi}\sum_{X=\pi,K}\left(\beta_B^{(X)} m_X + \beta_B^{\prime(X)} F(m_X,\delta,\mu)\right)$$
$$+\frac{1}{\Lambda_\chi^2}\sum_{X=\pi,K,\eta}\Bigg[\left(\gamma_B^{(X)} - \lambda_B^{(X)}\alpha_B + \frac{5}{2M_N}(\beta_B^{(X)} + \frac{1}{6}\beta_B^{\prime(X)})\right) m_X^2 \ln\frac{m_X^2}{\mu^2}$$
$$+\left(\tilde{\gamma}_B^{(X)} - \tilde{\lambda}_B^{(X)}\alpha_B\right) L_{(3/2)} + \hat{\gamma}_B^{(X)}\hat{L}_{(3/2)}\Bigg]\Bigg\}\,, \qquad (18)$$

where

$$\pi F(m,\delta,\mu) = -\delta\ln\tfrac{m^2}{\mu^2} + \begin{cases} 2\sqrt{m^2-\delta^2}\left(\frac{\pi}{2} - \arctan\left[\frac{\delta}{\sqrt{m^2-\delta^2}}\right]\right) & m \geq \delta \\ -2\sqrt{\delta^2-m^2}\ln\left[\frac{\delta+\sqrt{\delta^2-m^2}}{m}\right] & m < \delta \end{cases}$$

$$L_{(3/2)}(m,\delta,\mu) = m^2\ln\tfrac{m^2}{\mu^2} + 2\pi\delta F(m,\delta,\mu)$$
$$\hat{L}_{(3/2)}(m,\delta,\mu) = m^2\ln\tfrac{m^2}{\mu^2} + \tfrac{2\pi}{3\delta}G(m,\delta,\mu) \qquad (19)$$

$$\pi G(m,\delta,\mu) = -\delta^3\ln\tfrac{m^2}{\mu^2} + \pi m^3 - \begin{cases} 2(m^2-\delta^2)^{3/2}\left(\frac{\pi}{2} - \arctan\left[\frac{\delta}{\sqrt{m^2-\delta^2}}\right]\right) & m \geq \delta \\ 2(\delta^2-m^2)^{3/2}\ln\left[\frac{\delta+\sqrt{\delta^2-m^2}}{m}\right] & m < \delta. \end{cases}$$

Here μ is the scale of dimensional regularization whose value we take as 1 GeV in the following. The coefficients α_B are the tree level contributions which are linear combinations of the coupling constants appearing Eq. (3) and Eq. (17); $\beta_B^{(X)}$ and $\beta_B^{\prime(X)}$ are the contributions from the meson one-loop graphs in Figs 1a. containing intermediate octet and decuplet states, respectively; $\gamma_B^{(X)}$,$\tilde{\gamma}_B^{(X)}$ and $\hat{\gamma}_B^{(X)}$ are the contributions from the graphs in Fig. 1 b,c,d, respectively; $\lambda_B^{(X)}$ and $\tilde{\lambda}_B^{(X)}$ are the wavefunction renormalization contributions, again with octet and decuplet intermediate states, respectively. Due to our choice of normalizations, our coefficients appear different from those given in [2], [3] and [4] so we list them all in Appendix I. They are, however, in complete agreement with those references including the corrections noted in [4] and the Erratum to [2].

We turn now to a determination of the low-energy constants $b_\pm$, and b_i, $i = 3,\ldots,7$. It is instructive to consider the evolution of these constants as the chiral expansion is carried out to successively higher orders and as decuplet intermediate states are included. For the magnetic moments at $\mathcal{O}(1/\Lambda_\chi)$ and $\mathcal{O}(1/\Lambda_\chi^2)$, we perform an un-weighted least squares fit of the leading order constants $b_\pm$ using the seven well-measured octet magnetic moments (we exlude the μ_{Σ^0}). At $\mathcal{O}(1/\Lambda_\chi^3)$, there appear the five additional symmetry breaking constants plus the unknown constants $b_9 - b_{11}$. We follow the same procedure as Ref. [3] by using resonance saturation to determine $b_9 - b_{11}$. The details can be found in that reference. We note, however, that where we have taken the decuplet as an explicit degree of freedom

we do not include its contribution to these couplings. We chose to leave the symmetry breaking constants as fit parameters and use the seven well-measured moments to obtain an exact solution. The values of the b_i are given in Table I, for two scenarios: (O) – only the octet loop corrections included at a given order, and (O+D) – both octet and decuplet loop effects included. A measure of the quality of the fits is given in Table II, where the magnetic moments predicted at a given order are compared with the experimental values (final two columns). At $\mathcal{O}(1/\Lambda_\chi^3)$ only the $\Sigma^0-\Lambda$ transition moment is a prediction.

As observed in previous analyses, at $\mathcal{O}(1/\Lambda_\chi^2)$ the fit without the decuplet ($\chi^2=0.377$) is better than the one where it is included ($\chi^2=0.651$). Evidently, truncation of the chiral expansion at $\mathcal{O}(1/\Lambda_\chi^2)$ is not sufficient in this case. At $\mathcal{O}(1/\Lambda_\chi^3)$, the fits are exact and it is difficult to see the effect of the decuplet. To gain some insight we examine the contribution from each order individually. As an example we consider the magnetic moment of the proton. In the case where only intermediate octet states are considered the magnetic moment breaks down as follows

$$\mu_p = 3.268(1-.687+.541) = 2.791. \tag{20}$$

Here we have normalized to the tree level moment, which arises at $\mathcal{O}(1/\Lambda_\chi)$. The second and third terms in parentheses correspond to the $\mathcal{O}(1/\Lambda_\chi^2)$ and $\mathcal{O}(1/\Lambda_\chi^3)$ corrections, respectively. We see that the contribution from the $\mathcal{O}(1/\Lambda_\chi^3)$ terms are as large as those contributing at $\mathcal{O}(1/\Lambda_\chi^2)$. In the case where the decuplet is included we obtain

$$\mu_p = 4.695(1-.513+.108) = 2.793. \tag{21}$$

Here we see the effect of the decuplet. Naïvely, one would expect the corrections to scale as (p/Λ_χ) and $(p/\Lambda_\chi)^2$, respectively, relative to the tree-level contribution. Taking $p=m_k$, then one expects the various orders to contribute as $1:1/2:1/4$ or as $1:1/3:1/9$ using $p=\delta$. Clearly, this pattern does not obtain in the case of the octet only calculation, but does in the octet+decuplet case. We find a similar conclusion for each octet magnetic moment.

$$\begin{array}{rcrcr}
\mu_p &=& 4.695(1-.513+.103) &=& 2.793 \\
\mu_n &=& -3.204(1-.446+.043) &=& -1.913 \\
\mu_{\Xi^-} &=& -1.491(1-.703+.141) &=& -0.653 \\
\mu_{\Xi^0} &=& -3.204(1-.929+.319) &=& -1.250 \\
\mu_{\Sigma^+} &=& 4.695(1-.707+.230) &=& 2.450 \\
\mu_{\Sigma^-} &=& -1.491(1-.184-.039) &=& -1.160 \\
\mu_{\Lambda} &=& -1.601(1-.950+.332) &=& -0.613 \\
\mu_{\Sigma^0\Lambda} &=& 2.775(1-.600+.138) &=& -1.491
\end{array}$$

Apart from a few exceptions ($\mu_\Lambda,\ \mu_{\Xi^0}$ at $\mathcal{O}(1/\Lambda_\chi^2)$), the chiral expansion seems to converge as expected when the $\mathcal{O}(1/M_{\rm B})$ corrections and the decuplet are included explicitly. The result in Eq.(21) should be compared to that of Ref. [3]

$$\mu_p = 4.48(1-.49+.11) = 2.79. \tag{22}$$

Eqs.(21) and (22) are essentially identical, so we must make some comment on how they differ. Eq.(20) is the result of a calculation similar to the one made in Ref. [3] to obtain Eq.(22). They are different only in that we include only the non-analytic contributions from loops. In Ref. [3] some of the loops appearing at $\mathcal{O}(1/\Lambda_\chi^3)$ have the analytic structure

$$\text{constant} \times (m_X^2 \ln \frac{m_X^2}{\mu^2} - m_X^2)$$

where $X = \pi, K$. Evidently, the inclusion of the analytic piece cancels a significant portion of the non-analytic one. To be specific, for the pions the cancellation is about 25% and for kaons it is greater than 70%. Since the analytic piece of a loop (or any portion of it) can be absorbed into the counterterms, the prescription for retaining it explicitly is ambiguous. It is satisfying to see that we can obtain the expected behavior of the expansion without resorting to this procedure.

To exhibit the importance of including the $1/M_N$ corrections and the contribution from the double derivative terms we compare Eq.(21) to the result of Ref. [4]

$$\mu_p = 3.668(1 - .651 + .412) = 2.791, \tag{23}$$

Here again the contribution from the $\mathcal{O}(1/\Lambda_\chi^3)$ terms is a larger fraction of those from $\mathcal{O}(1/\Lambda_\chi^2)$ than expected.

In summary we have re-examined the calculation of the magnetic moments for the octet of spin-$\frac{1}{2}$ baryons to $\mathcal{O}(1/\Lambda_\chi^3)$ in HBχPT. We have included all terms which contribute to this order. The decuplet of spin-$\frac{3}{2}$ was included as an explicit degree of freedom and its contribution to the octet magnetic moments evaluated. Our analysis indicates that including the decuplet is necessary to insure the correct size of the the contributions from succeeding orders in the chiral expansion. Only the non-analytic contributions of loops were retained so we avoid the ambiguities involved in including any analytic pieces. We also find no need to take smaller values for the axial couplings. Thus, it appears that a well-behaved, consistent chiral expansion of the octet baryon magnetic moments is attainable at $\mathcal{O}(q^4)$.[1] This result should put the baryon magnetic moments on the same chiral footing as other low-energy baryon properties.

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[1] In this respect we also mention here the analysis of Ref. [10]. The authors of that Ref. [10] develop a regularization scheme in which a momentum cut-off is introduced to suppress short-distance contributions to the Feymann integrals. It appears that this procedure may improve the convergence of the chiral expansion for the magnetic moments while respecting the chiral symmetry.

REFERENCES

[1] See. *e.g.*, V. Bernard, N. Kaiser, U-G. Meißner, Z. Phys **C**60 (1993) 111; B.R. Holstein, *Comments Nucl. Part. Phys.* **20**, 301 (1992); V. Bernard, N. Kaiser, J. Kambor, U-G. Meißner, Nucl. Phys. **B388**, (1992) 301; B. Borasay, Eur. Phys. J **C8** (1999) 121; J. Bijnens, H.Sonoda, M.B. Wise, Nucl. Phys **B261**, (1999) 185.

[2] E. Jenkins, M. Luke, A.V. Manohar and M.Savage, Phys. Lett. **B** 302 (1993) 482; **B** 388 (1996) (E).

[3] Ulf-G. Meißner and S. Steininger, Nucl. Phys. **B** 499 (1997) 349.

[4] Loyal Durand and Phuoc Ha, Phys. Rev. **D58** (1998) .

[5] M. J. Ramsey-Musolf and Hiroshi Ito, Phys. Rev. **C** (1997) 2066.

[6] E. Jenkins and A.V. Manohar, Baryon chiral perturbation theory, in: Proc. Workshop on effective field theories of the standard model, ed. Ulf-G. Meißner (World Scientific, Singapore, 1992).

[7] E. Jenkins and A.V. Manohar, Phys. Lett. **B** 259 (1991) 353.

[8] M.K. Bannerjee and J. Milana, Phys. Rev. **D** 54 (1996) 5804.

[9] T. Hemmert, B.R. Holstein, J. Kambor, J. Phys. **G24** (1998) 1831.

[10] J.F Donoghue, B.R. Holstein, B. Borasoy, Phys. Rev. **D** 59 (1999) 36002.

TABLES

	$\mathcal{O}(1/\Lambda_\chi)$	$\mathcal{O}(1/\Lambda_\chi^2)$		$\mathcal{O}(1/\Lambda_\chi^3)$	
CT		O	O+D	O	O+D
b_+	1.490	2.999	3.606	1.994	2.989
b_-	1.098	2.194	2.194	1.368	1.924
b_3	–	–	–	−0.297	−0.327
b_4	–	–	–	0.277	−0.159
b_5	–	–	–	−0.086	0.124
b_6	–	–	–	0.576	0.171
b_7	–	–	–	−0.952	−1.166

TABLE I. Couplings for leading order and symmetry breaking magnetic moment counterterms at each order, with and without including the decuplet intermediate states in loops. "O" and "D" denote octet and decuplet respectively.

	$\mathcal{O}(1/\Lambda_\chi)$	$\mathcal{O}(1/\Lambda_\chi^2)$		$\mathcal{O}(1/\Lambda_\chi^3)$	
μ_{Fit}		O	O+D	O	O+D
p	2.564	2.890	3.051	2.793	2.793
n	−1.597	−2.360	−2.437	−1.913	−1.913
Ξ^-	−0.967	−0.585	−0.547	−0.651	−0.651
Ξ^0	−1.597	−0.933	−0.887	−1.250	−1.250
Σ^+	2.564	2.287	2.141	2.458	2.458
Σ^-	−0.967	−1.298	−1.321	−1.160	−1.160
Λ	−0.799	−0.494	−0.410	−0.613	−0.613
$\Sigma^0\Lambda$	1.383	1.617	1.6827	1.520	1.491

TABLE II. Calculated values of the magnetic moments using fit values of the counterm couplings. "O" and "D" denote octet and decuplet respectively.

Appendix I.

Here we tabulate the coeffcients appearing the expressions for the magnetic moments.

	α_B
p	$\frac{1}{3}b_+ + b_- + b_3 + b_4 + \frac{1}{3}b_5 + \frac{1}{3}b_6 - \frac{1}{3}b_7$
n	$-\frac{2}{3}b_+ - \frac{2}{3}b_5 - \frac{2}{3}b_6 - \frac{2}{3}b_7$
Ξ^-	$-\frac{1}{3}b_+ - b_- + b_3 - b_4 - \frac{1}{3}b_5 + \frac{1}{3}b_6 - \frac{1}{3}b_7$
Ξ^0	$\frac{-2}{3}b_+ + \frac{1}{3}b_5 - \frac{1}{3}b_6 - \frac{1}{3}b_7$
Σ^+	$\frac{1}{3}b_+ + 1b_- - \frac{1}{3}b_7$
Σ^-	$\frac{1}{3}b_+ - 1b_- - \frac{1}{3}b_7$
Λ	$-\frac{1}{3}b_+ - \frac{8}{9}b_6 - \frac{1}{3}b_7$
$\Sigma^0\Lambda$	$\frac{1}{\sqrt{3}}b_+$

	$\beta_B^{(X)}$	
	π	K
p	$-(D+F)^2$	$-(\frac{2}{3}D^2 + 2F^2)$
n	$(D+F)^2$	$-(D-F)^2$
Ξ^-	$(D-F)^2$	$(\frac{2}{3}D^2 + 2F^2)$
Ξ^0	$-(D-F)^2$	$(D+F)^2$
Σ^+	$-(\frac{2}{3}D^2 + 2F^2)$	$-(D+F)^2$
Σ^-	$(\frac{2}{3}D^2 + 2F^2)$	$(D-F)^2$
Λ	0	$2DF$
$\Sigma^0\Lambda$	$-\frac{4}{\sqrt{3}}DF$	$-\frac{2}{\sqrt{3}}DF$

	$\beta_B'^{(X)}$	
	π	K
p	$-\frac{2}{9}\mathcal{C}^2$	$\frac{1}{18}\mathcal{C}^2$
n	$\frac{1}{18}\mathcal{C}$	$\frac{1}{9}\mathcal{C}^2$
Ξ^-	$-\frac{1}{9}\mathcal{C}^2$	$-\frac{1}{18}\mathcal{C}^2$
Ξ^0	$\frac{1}{9}\mathcal{C}^2$	$\frac{2}{9}\mathcal{C}^2$
Σ^+	$\frac{1}{18}\mathcal{C}^2$	$-\frac{2}{9}\mathcal{C}^2$
Σ^-	$-\frac{1}{18}\mathcal{C}^2$	$-\frac{1}{9}\mathcal{C}^2$
Λ	0	$\frac{1}{6}\mathcal{C}^2$
$\Sigma^0\Lambda$	$-\frac{1}{3\sqrt{3}}\mathcal{C}^2$	$-\frac{1}{6\sqrt{3}}\mathcal{C}^2$

	$\gamma_B^{(\pi)}$
p	$-(b_+ + b_-) + \frac{1}{2}(D+F)^2(b_+ - b_-) + 2(b_{10} + b_{11})$
n	$b_+ + (1-(D+F)^2)\,b_- - 2(b_{10}+b_{11})$
Ξ^-	$b_- - b_+ + \frac{1}{2}(D-F)^2(b_+ + b_-) + 2(b_{10} - b_{11})$
Ξ^0	$b_+ - (1-(D-F)^2)\,b_- - 2(b_{10}+b_{11})$
Σ^+	$\frac{2}{9}(D^2+6DF-6F^2)b_+ - 2(1+F^2)b_- + 2(b_{10}+b_{11})$
Σ^-	$\frac{2}{9}\,(D^2-6DF-6F^2) + 2(1+F^2)b_- - b_9 - 4b_{11}$
Λ	$-\frac{2}{3}D^2b_+$
$\Sigma^0\Lambda$	$-\frac{2}{3\sqrt{3}}(3-D^2)b_+ + \frac{4}{3\sqrt{3}}DFb_- + \frac{4}{\sqrt{3}}b_{10}$

	$\gamma_B^{(K)}$
p	$-\left(\frac{1}{9}D^2 - 2DF + F^2\right)b_+ - (2+(D-F)^2)\,b_- + b_9 + 4b_{11}$
n	$\left(1 - \frac{7}{9}D^2 + \frac{2}{3}DF + F^2\right)b_+ - (1-(D-F)^2)\,b_- + 2(b_{11} - b_{10})$
Ξ^-	$-\left(\frac{1}{9}D^2 + 2DF + F^2\right)b_+ + (1+(D+F)^2)\,b_- - b_9 - 4b_{11}$
Ξ^0	$\left(1 - (\frac{7}{9}D^2 + \frac{2}{3}DF - F^2)\right)b_+ + (1-(D+F)^2) - 2(b_{10}+b_{11})$
Σ^+	$\left((\frac{1}{3}D^2 + 2DF + \frac{1}{3}F^2) - 1\right)b_+ - (1+(D-F)^2)\,b_- + 2(b_{10}+b_{11})$
Σ^-	$\left((\frac{1}{3}D^2 - 2DF + \frac{1}{3}F^2) - 1\right)b_+ - (1+(D+F)^2)\,b_- + 2(b_{10}-b_{11})$
Λ	$\left(1 + \frac{1}{9}D^2 + F^2\right)b_+ - 2DFb_- - 2b_{10}$
$\Sigma^0\Lambda$	$\frac{1}{\sqrt{3}}\,(D^2 - 3F^2 - 1)\,b_+ + \frac{2}{\sqrt{3}}DFb_- + \frac{4}{\sqrt{3}}b_{10}$

	$\gamma_B^{(\eta)}$
p	$-\frac{1}{18}(D-3F)^2(b_+ + 3b_-)$
n	$\frac{1}{9}(D-3F)^2 b_+$
Ξ^-	$\frac{1}{18}(D+3F)^2(b_+ - 3b_-)$
Ξ^0	$\frac{1}{9}(D+3F^2)b_+$
Σ^+	$-\frac{2}{9}D^2(b_+ + 3b_-)$
Σ^-	$-\frac{2}{9}D^2(b_+ - 3b_-)$
Λ	$\frac{2}{9}D^2 b_+$
$\Sigma^0\Lambda$	$\frac{2}{3\sqrt{3}}D^2 b_+$

	$\tilde{\gamma}_B^{(X)}$		
	π	K	η
p	$\frac{20}{27}\tilde{\mu}_{\rm C}\mathcal{C}^2$	$\frac{5}{54}\tilde{\mu}_{\rm C}\mathcal{C}^2$	0
n	$-\frac{5}{27}\tilde{\mu}_{\rm C}\mathcal{C}^2$	$-\frac{5}{54}\tilde{\mu}_{\rm C}\mathcal{C}^2$	0
Ξ^-	$-\frac{5}{108}\tilde{\mu}_{\rm C}\mathcal{C}^2$	$-\frac{10}{27}\tilde{\mu}_{\rm C}\mathcal{C}^2$	$-\frac{5}{36}\tilde{\mu}_{\rm C}\mathcal{C}^2$
Ξ^0	$-\frac{5}{54}\tilde{\mu}_{\rm C}\mathcal{C}^2$	$-\frac{5}{27}\tilde{\mu}_{\rm C}\mathcal{C}^2$	0
Σ^+	$\frac{5}{108}\tilde{\mu}_{\rm C}\mathcal{C}^2$	$\frac{35}{54}\tilde{\mu}_{\rm C}\mathcal{C}^2$	$\frac{5}{36}\tilde{\mu}_{\rm C}\mathcal{C}^2$
Σ^-	$-\frac{5}{108}\tilde{\mu}_{\rm C}\mathcal{C}^2$	$-\frac{10}{27}\tilde{\mu}_{\rm C}\mathcal{C}^2$	$-\frac{5}{36}\tilde{\mu}_{\rm C}\mathcal{C}^2$
Λ	0	$-\frac{5}{36}\tilde{\mu}_{\rm C}\mathcal{C}^2$	0
$\Sigma^0\Lambda$	$\frac{5}{18\sqrt{3}}\tilde{\mu}_{\rm C}\mathcal{C}^2$	$\frac{5}{36\sqrt{3}}\tilde{\mu}_{\rm C}\mathcal{C}^2$	0

	$\hat{\gamma}_B^{(X)}$		
	π	K	η
p	$\frac{4}{9}(D+F)\mathcal{C}\tilde{\mu}_{\rm T}$	$\frac{1}{9}(3D-F)\mathcal{C}\tilde{\mu}_{\rm T}$	0
n	$-\frac{4}{9}(D+F)\mathcal{C}\tilde{\mu}_{\rm T}$	$-\frac{2}{9}F\mathcal{C}\tilde{\mu}_{\rm T}$	0
Ξ^-	$\frac{2}{9}(F-D)\mathcal{C}\tilde{\mu}_{\rm T}$	$\frac{1}{9}(F-D)\mathcal{C}\tilde{\mu}_{\rm T}$	0
Ξ^0	$\frac{2}{9}(F-D)\mathcal{C}\tilde{\mu}_{\rm T}$	$\frac{2}{9}(D+2F)\mathcal{C}\tilde{\mu}_{\rm T}$	$-\frac{1}{9}(D+3F)\mathcal{C}\tilde{\mu}_{\rm T}$
Σ^+	$\frac{1}{9}(D+3F)\mathcal{C}\tilde{\mu}_{\rm T}$	$\frac{4}{9}D\mathcal{C}\tilde{\mu}_{\rm T}$	$\frac{2}{9}D\mathcal{C}\tilde{\mu}_{\rm T}$
Σ^-	$\frac{1}{9}(F-D)\mathcal{C}\tilde{\mu}_{\rm T}$	$\frac{2}{9}(F-D)\mathcal{C}\tilde{\mu}_{\rm T}$	0
Λ	$-\frac{1}{3}D\mathcal{C}\tilde{\mu}_{\rm T}$	$\frac{1}{9}(D-3F)\mathcal{C}\tilde{\mu}_{\rm T}$	0
$\Sigma^0\Lambda$	$\frac{1}{18\sqrt{3}}(D+6F)\mathcal{C}\tilde{\mu}_{\rm T}$	$\frac{2}{9\sqrt{3}}(2D+3F)\mathcal{C}\tilde{\mu}_{\rm T}$	$\frac{1}{6\sqrt{3}}D\mathcal{C}\tilde{\mu}_{\rm T}$

	$\lambda_B^{(X)}$		
	π	K	η
N	$\frac{9}{4}(D+F)^2$	$\frac{5}{2}D^2 - 3DF + \frac{9}{2}F^2$	$\frac{1}{4}(D-3F)^2$
Ξ	$\frac{9}{4}(D-F)^2$	$\frac{5}{2}D^2 + 3DF + \frac{9}{2}F^2$	$\frac{1}{4}(D+3F)^2$
Σ	$D^2 + 6F^2$	$3(D^2 + F^2)$	D^2
Λ	$3D^2$	$D^2 + 9F^2$	D^2
$\Sigma^0\Lambda$	$2D^2 + 3F^2$	$2D^2 + 6F^2$	D^2

	$\tilde{\lambda}_B^{(X)}$		
	π	K	η
N	$2\mathcal{C}^2$	$\frac{1}{2}\mathcal{C}^2$	0
Ξ	$\frac{1}{}\mathcal{C}^2$	$\frac{3}{2}\mathcal{C}^2$	$\frac{1}{2}\mathcal{C}^2$
Σ	$\frac{1}{3}\mathcal{C}^2$	$\frac{5}{3}\mathcal{C}^2$	$\frac{1}{2}\mathcal{C}^2$
Λ	$\frac{3}{2}\mathcal{C}^2$	$\mathcal{C}^2$	0
$\Sigma^0\Lambda$	$\frac{11}{12}\mathcal{C}^2$	$\frac{4}{3}\mathcal{C}^2$	$\frac{1}{4}\mathcal{C}^2$